Simulating Electron Scattering in Cold-Cathode Discharges

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Abstract

Features of EDIPIC particle-in-cell simulation code

Study of anisotropic electron velocity distributions in cathode fall and negative glow

Approximation of angular scattering in electron-neutral elastic collisions for use in simulations
Abstract

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We have studied the formation of electron velocity distribution function (EVDF) in different parts of the energy range in a short glow discharge in light gases. Models of such discharges are simplified by the fact that cross sections of elastic scattering is not large compared with the ionization.

*We show that the EVDF can be effectively described by splitting the electron population in two parts.*

The EVDF for most of the electrons produced at the beginning of the cathode layer is highly anisotropic [1], and remains so as they propagate into the negative glow. This group can be described by a 1D EVDF, with scattering accounted by small corrections. Another group of non-thermal electrons, originating inside the cathode fall, with lower energies becomes close to isotropic due to faster scattering collisions. This group can be treated under a diffusion approximation. Both groups are strongly non-Maxwellian. We interpret simulation results obtained with a particle-in-cell code EDIPIC.

EDIPIC (1D electrostatic parallel direct implicit particle-in-cell)

Direct implicit
Parallel
Coulomb collisions (electron-electron and electron-ion) accounted for
Ion and electron induced secondary electron emission
Resolves sheath and plasma simultaneously
Extensively validated
Extensive set of kinetic diagnostics for EVDF
Need for improved treatment of electron scattering

EVDF anisotropic $\Rightarrow$ good MC model of angular scattering is required, with a probability distribution easy to invert. Actual $d\sigma/d\Omega$ is a complicated function of $E, \theta$. Commonly used model [Surendra et al., 1990] has limited applicability:

$$\frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{1}{4\pi} \frac{E}{\left[1 + E \sin^2(\theta/2)\right] \ln(1 + E)}$$

with $E$ is in eV. The scattering angle is sampled according to

$$\theta = \cos^{-1} \left[ 1 + 2 \frac{1 - (1 + E)^R}{E} \right]$$

where $R$ is a random number from a uniform distribution on [0,1]. This approximation is convenient but *ad hoc* (not physical).
Model of electron scattering in He for use in Monte Carlo simulations

Use model of screened Coulomb potential and allow the screening parameter to be energy-dependent (Fernandez-Varea et al. [1992], Belenguer and Pitchford [1999], Okhrimovsky et al. [2002]):

\[ I(\theta) = \frac{1}{\sigma} \frac{d\sigma}{d\Omega} = \frac{1}{4\pi} \frac{1 - \xi^2(E)}{[1 - \xi(E)\cos\theta]^2}, \]

and \( \xi(E) \) is found by fitting to the ratio of total-to-transport cross-sections data from experiment or accurate theoretical calculations.

\[ \frac{1 - \xi}{2\xi^2} \left( (1 + \xi) \ln \frac{1 + \xi}{1 - \xi} - 2\xi \right) = \frac{\sigma_m}{\sigma}. \]

This approach automatically uses correct total and transport cross-sections and captures the main features of the actual \( d\sigma/d\Omega \) at different energies.
Proposed model for $e$-He scattering

For He gas, the solution $\xi(E)$ allows a 2-pole approximation:

$$\xi(E) = 1 + \frac{p_1 x - p_2}{(x - p_2)^2 + p_3} - \frac{p_1 x}{(x - p_4)^2 + p_5}, \quad x = \sqrt{E}[\text{eV}]$$

$p_1 = 2.45$, $p_2 = 2.82$, $p_3 = 11.98$, $p_4 = 5.11$, $p_5 = 64.01$

The graph shows a comparison of the approximated ratio to the one obtained from the data. The error in $\sigma_m/\sigma$ is within 1% in the fitting range $0 < E < 1000$ eV, and asymptotic behavior at $E \to \infty$, $\xi = 1 - A/E$, agrees with the theory. As usual, given a random number $0 < R < 1$, the scattering angle is sampled according to

$$\cos \theta = 1 - \frac{2R(1 - \xi(E))}{1 + \xi(E)(1 - 2R)}$$
Comparing $d\sigma/d\Omega$ for helium against data and previous model low electron energy
Comparing $d\sigma/d\Omega$ for helium against data and previous model

E=30 eV

$E=30$ eV, weighted with $\sin\theta$

Actual probability density sampled in MC simulation

Error at small angles is not important
EVDF map for electrons originating in the cathode fall shows complex structure

Using EDIPIC to simulate cold cathode DC discharge in helium

\[ U = 800 \text{ V}, \ p = 3 \text{ Torr}, \ L = 12 \text{ mm} \]

EVxDF, a.u.
Anisotropic velocity distribution in the cathode fall, showing the primary beam and the avalanche.

Secondary electrons scatter as they accelerate, but their distribution remains anisotropic.
EVDF <100eV is isotropic >200eV is a forward beam

Anisotropy parameter, as obtained from EVDF as a function of total energy and position. Near cathode there is a beam, so $a \to 0$; in the negative glow $a \to 1$. There is strong anisotropy in the entire cathode sheath at energies > 100 eV.

$$a = \frac{3}{2} \left\langle \sin^2 \theta \right\rangle$$

$a=1$: isotropic

$a<0.5$: forward beam
The EDIPIC code resolves potential variations to 0.1 V:

**Negative glow**

- Potential in the negative glow region
- Electron density
- Ion density
- Distance from cathode, m
- Potential, V
EVDF in negative glow: non-Maxwellian intermediate group and cathode beam

Electrons with energies < 100 eV, originating near the negative glow boundary, are scattering in NG and diffusing towards the anode. The distribution is isotropic but distinctly non-Maxwellian.

EVDF is constant along the lines of constant kinetic energy.
High energy part of the EVDF in cathode fall shows little scattering

Acceleration and multiplication is essentially a 1D process. EVDF is a function of total energy of parallel (to the electric field) motion.

\[
e_x = \frac{mv_x^2}{2} - e\phi(x)
\]
Comparison with analytical models

\[
f(\varepsilon) = \frac{\alpha m_e \Gamma_{\varepsilon}}{eE(x_0(\varepsilon))} \exp(\alpha x_0(\varepsilon)), \quad \alpha = n_0 \sigma_{\text{eff}}
\]

Integral equation with actual ionization $\sigma$, no scattering

Model: actual ioniz. x-section, no scattering
Kolobov-Tsendin, constant stopping power
Conclusions

Analytical approximation, suitable for Monte-Carlo simulations, of differential cross-section of electron scattering in He is obtained for wide range of energies (0.01-1000 eV).

The EVDF of the cathode fall in DC discharge is found from particle-in-cell simulations to be highly anisotropic for energies >200eV.

This group can be described by a one-dimensional EVDF, with small corrections for scattering.

For energies <100 eV, the EVDF is close to isotropic due to faster scattering.

This group can be treated making use of the diffusion approximation.