Transport in a field aligned magnetized plasma and neutral gas boundary:

The end of the plasma...

C.M. Cooper, W. Gekelman, P. Pribyl, J. Maggs, Z. Lucky
University of California, Los Angeles
February 15, 2013
Overview

• Motivation
  – Relation to **Auroral Physics, Divertor Physics**

• Summary of Enormous Toroidal Plasma Device
  – Model of **ETPD plasma regions**

• **Neutral Boundary Layer** model: Plasma Termination
  1. Occurs at **plasma/neutral gas pressure equilibrium**
  2. Plasma terminates on **current-free ambipolar sheath** determined by a generalized Ohm’s law
  3. **Heating, ionization** occur in NBL
     ➢ **Measured scaling laws** in ETPD
Motivation: Aurora

The auroral-arc current loop associated with a perpendicular electric field imposed in the magnetosphere [Borovsky 1993]

Aurora over Jokulsarlon Lake, Iceland
Stephane Vetter, “Nuits sacrees” Feb. 2011

Schematic view of parallel currents $j_{||}$, plasma motions, $v_{\text{perp}}$, electric fields, and potential contours above auroral arc, sketched in green [Haerendel 1996]

Motivation: Gaseous Divertors

Gaseous divertors designed to dissipate plasma particles, momentum, and energy on neutral gas before touching wall.

Diagram of the ETPD

Toroidal Magnets (red) provide a 250 G confining field

Vertical Magnets (blue) provide a 6 G vertical field to space out rings

LaB$_6$ source injects primaries to form then heat the plasma

ETPD Plasma (pink) follows the helical magnetic field up to 120 m

C.M. Cooper et al, Rev of Sci Inst. 81 083503 (2010)
Picture of a LaB$_6$ Cathode

Anode

Frame and Shielding

Heater

Cathode
Experimental Setup

• Primary electrons boiled off LaB$_6$ are accelerated along the field by anode.

• Primaries ionize neutral gas to create and heat plasma.

• Data taken where plasma ends, 90% around machine.
Neutral Boundary Layer

400 V discharge 2 mTorr

300 V discharge 2.5 mTorr

220 V discharge 3.6 mTorr

Probe

<table>
<thead>
<tr>
<th>Discharge Voltage (Primary Energy)</th>
<th>Discharge Current (Primary Flux)</th>
<th>Toroidal B-Field</th>
<th>Neutral Gas</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 V</td>
<td>0 A</td>
<td>2.2 kA</td>
<td>0.8 mTorr</td>
</tr>
<tr>
<td>400 V</td>
<td>250 G</td>
<td>150 G</td>
<td>5.0 mTorr</td>
</tr>
</tbody>
</table>
Energy Balance in ETPD

Helium Gas

Ionosphere

Atmosphere

Energy Balance in Aurora
### 3 Zones in ETPD Plasma

<table>
<thead>
<tr>
<th>Heat</th>
<th>Density</th>
</tr>
</thead>
<tbody>
<tr>
<td>+ Thermalization of primaries</td>
<td>+ ionization losses</td>
</tr>
<tr>
<td>- Conduction/ionization</td>
<td>- Radial diffusion</td>
</tr>
<tr>
<td>- Ohmic Heating and cooling</td>
<td>- Ionization and Radial diffusion</td>
</tr>
</tbody>
</table>

- Energy input length: $\lambda_{mfp,\text{prim}} \sim 15m$
- Energy loss length: $\Gamma_p / \nabla_r \Gamma_r \sim 15m$
- Efield Term: $\nu_{in} / \nu_p \sim 1.5m$

Take Data
Ambipolar Termination Sheath in NBL!

Plasma potential in neutral gas boundary indicative of large field-aligned electric field

Neutral fill of 3.6 mTorr He, a plasma discharge of 194 A 220 V, Btor=250 G, Bver=6 G,
Generate Radial Profiles

Used to study rate at which particles, momentum, and heat move across the field and out of the plasma.

Neutral fill of 3.6 mTorr He, a plasma discharge of 194 A 220 V, Btor=250 G, Bver=6 G,
Generate Axial Profiles in NBL

Plot data along the center of plasma coordinate “s”

Three zones!

Neutral fill of 3.6 mTorr He, a plasma discharge of 194 A 220 V, Btor=250 G, Bver=6 G,
Neutral Boundary Layer Model

- **Three-fluid, current-free**

<table>
<thead>
<tr>
<th>Equation Type</th>
<th>Description</th>
<th>Mathematical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Continuity equation</td>
<td>Ionization and losses</td>
<td>[ \frac{\partial}{\partial s} \left(n_p v_{p,s}\right) = S_p - \nabla_\perp \Gamma_{p,\perp} \quad \frac{\partial}{\partial s} \left(n_n v_{n,s}\right) = -S_p - \nabla_\perp \Gamma_{n,\perp} ]</td>
</tr>
<tr>
<td>Momentum equation</td>
<td>Pressure equilibration</td>
<td>[ \frac{\partial}{\partial s} \left(p_p + p_n\right) = -m_i \left(v_{p,s} \nabla_\perp \Gamma_{p,\perp} + v_{n,s} \nabla_\perp \Gamma_{n,\perp}\right) \neq 0 ]</td>
</tr>
<tr>
<td>Momentum equation</td>
<td>Generalized Ohms Law</td>
<td>[ eE_s = -\frac{\partial p_e}{\partial s} - (0.71 + t_n)n_p k_B \frac{\partial T_e}{\partial s} - m_e n_p \left(v_{p,s} - v_{n,s}\right) v_{en} ]</td>
</tr>
<tr>
<td>Heat equation</td>
<td>Ionization and Ohmic heating</td>
<td>[ \frac{3}{2} n_p v_{p,s} k_B \frac{\partial T_e}{\partial s} = en_p v_{p,s} E_s - E_{ion} S_p ]</td>
</tr>
</tbody>
</table>
**Zone A: Pressure Equilibration**

For $p_p \geq p_n$

- Diffusivity argument: Neutral density convected out along field by plasma, diffuses across field

$$D_{a,||} \gg D_n = \frac{k_B T_n}{\nu_n} > D_{a,\perp}$$

- Drag force limited to cross-field neutral diffusion rate

$$\nu_{drag} = \nabla_\perp \Gamma_{n,\perp}/n_n \leq D_n/R_p^2$$

- Axial neutral flow develops from force balance

$$\nu_{n,s} = \nu_{p,s} \frac{(0.5\nu_{in} - \nu_{cx})/n_n}{(0.5\nu_{in} - \nu_{cx})/n_n + \nabla_\perp \Gamma_{n,\perp}/n_n}$$

Momentum gained from plasma

Momentum lost to stationary neutrals
Electron Temperature in **Zone A**

- Constant $T_e$
- Ionization $\sim 0$

\[ \lambda_{\Delta T_e} = \frac{m_i}{m_e} \lambda_{en} \sim 100 \text{ m} \]

\[ T_e = 2.2 \text{ eV} \]
Plasma Velocity in Zone A

- Plasma velocity is flat
- Momentum balance
  \[
  \frac{\partial p_e}{\partial s} = -m_i v_{p,s} \nabla \Gamma_{p,r} + m_i n_e (v_{p,s} - v_{n,s})(0.5v_{in} + v_{cx})
  \]
- Sets a critical velocity
  \[
  v_{eq,s} = c_s \sqrt{\frac{\nabla \Gamma_{p,r}}{\nabla \Gamma_{p,r} + \nabla \Gamma_{n,r}}}
  \]
Plasma Density in **Zone A**

- Plasma density is dropping from radial losses

\[
\nu_{p,s} \frac{\partial n_p}{\partial s} = -\nabla_r \Gamma_{p,r} = -D_r \frac{n_p}{\lambda_{\perp}^2}
\]

- \[ D_r \sim 4D_{a,\perp} \sim \frac{1}{2} D_{Bohm} \]
- \[ \lambda_{\perp} \sim 10 \text{ cm} \]
Zone B: Ambipolar Electric Field

**Current-free** plasma sees neutral gas at end

Electrons stop, **ion current** penetrates

Electric field develops, drives **electron current**

Electric field maintains **Current free** term.
Potential Structure in NBL

- Plasma iso-potentials form “nested V’s”
- White arrows show electric field, parallel field 10x
- **Quasineutral**

Colormap of plasma potential from 1,300 points interpolated onto toroidal coordinates
Electron Temperature in Zone B

- Electron temperature rises with potential
- For
  \[ \lambda_{en} < L_{ΔΦ} < \frac{m_e}{m_i} \lambda_{en} \]

\[ ΔT_e \sim ΔΦ_p \sim 1 \text{eV} \]
Plasma Velocity in **Zone B**

- Plasma velocity drops due to drag on neutrals

**Plasma Velocity**

- **$\Phi_p$** (eV)
- **$V_p$** ($10^5$ cm/s)

**Graphical Data:**
- Distance from anode (cm)
- Plasma velocity at 2.2 eV

![Graph showing plasma velocity and other data](image-url)
The difference between momentum and temperature loss

<table>
<thead>
<tr>
<th>Electrons</th>
<th>Ions</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Momentum loss:</strong></td>
<td><strong>Momentum loss:</strong></td>
</tr>
<tr>
<td>( \nu_{en} - \text{faster} )</td>
<td>( .5\nu_{in} + \nu_{cx} - \text{slower} )</td>
</tr>
<tr>
<td><strong>Energy loss:</strong></td>
<td><strong>Energy loss:</strong></td>
</tr>
<tr>
<td>( m_i \nu_{en} - \text{slower} )</td>
<td>( .5\nu_{in} + \nu_{cx} - \text{faster} )</td>
</tr>
</tbody>
</table>

In electric field, electrons **HEAT.**

In electric field, ions **SLOW.**
Plasma Density in **Zone B**

- Plasma density is dropping from radial losses

- Balance between Flux conservation
  \[ \downarrow n_p \implies \uparrow v_{p,s} \]

- Adiabatic heating
  \[ \uparrow T_e \implies \downarrow n_p \]
Plasma Termination

Use generalized Ohms law to solve for electric field

\[ 0 = -\frac{\partial p_e}{\partial s} - e n_p E - (0.71 + t_n) n_p \frac{\partial T_e}{\partial s} - m_e n_p v_{p,s} v_{en} \]

\[ E \approx -\frac{m_e v_{p,s} v_{en}}{0.19e} \approx \frac{2V}{m} \text{ for } t_n = 0.1 \]

\[ E = \frac{J_e}{\alpha \sigma_{en}} = -\frac{J_i}{\alpha \sigma_{en}}, \quad J_{tot} = 0 \]
Zone C: Additional Ionization

\[ T_e = 2.2 \text{eV} \]

Electron Temperature in Zone C

- Prediction: Electron temperature continues to rise rapidly with potential.
Electron Temperature in **Zone C**

\[ \Phi_p - 2.5 \text{ eV} \]

Ionization Processes in Helium

\[ \langle \sigma v \rangle \approx 4.7 \text{ eV} \]

\[ \langle \sigma v \rangle \approx 2.2 \text{ eV} \]

~1000
Plasma Density in **Zone C**

- **Prediction**: Plasma density continues to drop.

![Graph showing plasma density and distance from anode](image)

- $\Phi_p$ (eV)
- $n_e$ ($10^{12}$/cm$^3$)

Distance from anode (cm)
Plasma Density in **Zone C**

![Graph showing plasma density and ionization with specific values]
Plasma Production in Boundary

If all energy gained went into plasma production how much could you make? For \( T_b = T_a, \ v_b = v_a \)

\[
Q_{\Delta \Phi} = -Q_{\text{ionz}}
\]

\[
\frac{\Delta n_e}{n_e} = \frac{\Delta \Phi}{E_{\text{ionz}} + T_e} = \frac{2.5 eV}{24.6 eV + 2.2 eV} \sim 10\%
\]

What’s the mean free path of ionization?

\[
\frac{v_{pz}}{n_n \langle \sigma v \rangle |_{4.7 eV}} \sim 10 \text{ cm}
\]

Any energy gained by electric field is quickly absorbed by ionization
Plasma Velocity in Zone C

- **Prediction**: Plasma velocity will drop to 0
Plasma Velocity in **Zone C**

- **IONIZATION**
  - Additional flux is generated by the ionization at the end of the plasma
  - Still slowing down after ionization ends

![Graph showing plasma velocity vs. distance from anode](image-url)
NBL Conserved Quatities

Normalized Plasma Pressure:

- End of the plasma occurs where $p_p = p_n$
- Dropping pressure “interrupted” by termination E field

Axial Plasma Flux:

Plasma flux:
- Measure of axial particle flux
- Only loss of axial flux is radial flux
- Filling in of profile
- Kinetic effects (trapped particles)
The potential marks a "footprint" for the boundary of the plasma.

The location and magnitude of Electric field terminating the plasma change as a function of input power.
Scaled Plasma Parameters pre-NBL

- **Te is flat** at 2.2 eV
- Extra power raises density
- Axial flow drops
Scaling Study

(a) Electric field scaling in NBL

Measured electric field, $E_{s,m}$, scales like the theoretical ambipolar value

$$E_{s,t} \approx -\frac{m_e v_{p,s} \nu_{en}}{0.19e}$$

(b) Termination point scaling in NBL

Measured termination point, $s_{eq,m}$, coincides with location of pressure equilibration

$$s_{eq,t} = \frac{v_{p,s} R_p^2}{D_r} \ln \left( \frac{(T_e + T_i) n_p (s_0)}{T_n n_n} \right)$$
Modified Ambipolar Flow in ETPD
Ambipolar Flow in NBL

- Magnetic field data from probe indicates axial current
- Negative current carried by electrons moving in positive direction

Data taken at $s = 2870$ cm, halfway through NBL
Non-Zero Current in NBL

- NBL not current free
- Current trends to zero
- Drift speed associated with current $\ll$ bulk flow

\[ \Delta \Delta \Delta J_{T,\text{meas}} = \nabla \times B_{\text{meas}} \]

\[ 0.9 v_{e,s} < v_{i,s} < v_{e,s} \]
### Types of Current

Some currents diverge and need to be closed to maintain $\nabla \cdot J = 0$.

<table>
<thead>
<tr>
<th>Divergence-Free</th>
<th>Non Divergence-Free</th>
</tr>
</thead>
<tbody>
<tr>
<td>• Poloidal Hall (i-n) Current (+)</td>
<td>• Parallel (e-n) Current</td>
</tr>
<tr>
<td>• Radial Pederson (i-n) Current</td>
<td>• Vertical $/nB$ Currents</td>
</tr>
<tr>
<td>• Poloidal Diamagnetic Current (-)</td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Steady state so $J_{polz} = 0$

Some currents are associated with particle drifts and transport.

- **Transport Causing**
  - Poloidal Hall (i-n) Current (+)
  - Radial Pederson (i-n) Current
  - Vertical $/nB$ Currents

- **Non Transport Causing**
  - Poloidal Diamagnetic Current (-)
Estimation of $J_r$

- **Axial** current tied to **Radial** current

Kirchhoff’s Junction Rule

$$J_{So+DS} = J_{So} + \frac{A_r}{A_S} J_r$$

$$E_r^* = E_r - \frac{\nabla_r p_r}{n_e} = m_i v_{ir} v_{in}$$

$$J_{ir} = \sigma_{in,\perp} E_r^*$$
Future Work on NBL

• Observational
  – Measure plasma potential and flows in Aurora

• Experimental
  – Study NBL for different pressures and/or gases

• Simulation
  – Model in DEGAS (neutral gas collisional code)*
  – Model in UEDGE (transport code)*
  – Runge Kutta solver for fluid model**
Future work: 1.5-D Simulation

Measured 5 things: $n_e, T_e, v_i, \phi_p, J_p$

Model neutral gas

Solve 5 equations + neutral gas transport:

\[
\begin{align*}
    n_e \frac{\partial v_{ez}}{\partial z} + v_{ez} \frac{\partial n_e}{\partial z} &= S_{en} \\
    n_e \frac{\partial v_{iz}}{\partial z} + v_{iz} \frac{\partial n_e}{\partial z} &= S_{in} \\
    \frac{3}{2} n_e v_{ez} \frac{\partial T_e}{\partial z} &= -\frac{\partial q_e}{\partial z} - n_e T_e \frac{\partial v_{ez}}{\partial z} + S_{eT} \\
    n_e v_{ez} \frac{\partial v_{ez}}{\partial z} &= -T_e \frac{\partial n_e}{\partial z} + \Xi \frac{\partial T_e}{\partial z} + n_e \frac{\partial \phi_p}{\partial z} + \Sigma_{ev} \\
    n_e v_{iz} \frac{\partial v_{iz}}{\partial z} &= -T_i \frac{\partial n_e}{\partial z} - n_e \frac{\partial \phi_p}{\partial z} + \Sigma_{iv}
\end{align*}
\]

Generalized Source/Sink calibrated by data
Conclusions

– Plasma terminates on **ambipolar electric field**
– Electric field occurs where diminishing **plasma pressure and neutral pressure equilibrate**
– NBL dominated by termination field through **Ohmic heating and ionization**
– NBL has a **small modified ambipolar diffusion** due to differences in directional particle motilities
Thanks!