Collective Focusing of a Neutralized Intense Ion Beam Propagating Along a Weak Solenodial Magnetic Field

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Motivation: Controlled Fusion

How can we achieve controlled fusion?

three main ways

- gravitational confinement
  
  “a day without fusion is like a day without sunshine”

- magnetic confinement
  
  “…like holding jello together with rubber bands” - Edward Teller

- inertial confinement
  
  “A small supernova. Very small” - Ed Moses

<table>
<thead>
<tr>
<th></th>
<th>density</th>
<th>temperature</th>
<th>confinement time</th>
<th>status</th>
</tr>
</thead>
<tbody>
<tr>
<td>gravitational</td>
<td>$10^4$ x solid</td>
<td>1 keV</td>
<td>$10^5$ years</td>
<td>proven daily</td>
</tr>
<tr>
<td>magnetic</td>
<td>$10^{-8}$ x solid</td>
<td>10 keV</td>
<td>seconds</td>
<td>first test 2020</td>
</tr>
<tr>
<td>inertial</td>
<td>$10^3$ x solid</td>
<td>10 keV to ignite</td>
<td>10’s of picoseconds</td>
<td>first test 2011</td>
</tr>
</tbody>
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Inertial Confinement Fusion: Lasers Versus Ion Beams

Present approach – Laser Driven ICF

Promising alternative – Heavy Ion Fusion

Why ion beams?

- High production efficiency and repetition rate
- High efficiency of energy delivery and deposition
Heavy Ion Fusion (Future)

- $I_{\text{total}} \sim 100$ kA
- $\tau_b \sim 10$ ns
- $E_b \sim 10$ GeV
- Atomic mass $\sim 200$

$\rho \sim 1000$ g/cm$^3$
$T \sim 10$ KeV

Warm Dense Matter Physics (Present)

- $I_b \sim 1-10$ A
- $\tau_b \sim 1$ ns
- $E_b \sim 0.1$-1 MeV
- Ions: $K^+$, $Li^+$

Presently accessible $\rho$-$T$ regime

$\rho \sim 1$ g/cm$^3$, $T \sim 0.1 \div 1$ eV

corresponds to the interiors of giant planets and low-mass stars

Block scheme of an ion driver for high energy density physics

- ion source
- acceleration and transport
- neutralized compression
- final focusing
- target
Neutralized Drift Compression Experiment (NDCX)

Heavy ion driver for Warm Dense Matter Experiments

Built and operated at the Lawrence Berkeley National Laboratory
Neutralized Drift Compression Experiment (NDCX)

Schematic of the NDCX-I experimental setup (LBNL)

Beam parameters at the target plane

K$^+ @ 300$ keV ($\beta_b=0.004$)

\[ I_b \sim 2 \text{ A} , r_b < 5 \text{ mm} \ (n_b \sim 10^{11} \text{ cm}^{-3}) \]

\[ T_{\text{target}} \sim 0.1 \text{ eV} \]

Upgrade NDCX-II

Li$^+ @ 3$ MeV ($\beta_b=0.03$)

\[ I_b \sim 30 \text{ A} , r_b \sim 1 \text{ mm} \ (n_b \sim 6 \cdot 10^{12} \text{ cm}^{-3}) \]

\[ T_{\text{target}} \sim 1 \text{ eV} \]

Weak fringe magnetic fields ($\sim 100$ G) penetrate deeply into the background plasma
I. Enhanced self-focusing of an ion beam propagating through a background plasma along a weak (~100 G) solenodial magnetic field

- important for the design of a heavy-ion driver (e.g. NDCX neutralized drift section)
- can be utilized in ion beam self-pinch transport applications (e.g. HIF drivers)

II. Collective Focusing (Robertson) Lens

- can be used for the ion beam final focus (e.g. NDCX-I, II)
- perhaps can be utilized for collimation of laser generated proton beams

Collective focusing with $B_0 \sim 1$ kG is equivalent to standard magnetic focusing with $B_0 \sim 10$ T
I. Ion Beam Propagation through a Neutralizing Background Plasma Along a Solenoidal Magnetic Field
Magnetic Self-Pinching ($B_0=0$)

- The ion beam space-charge is typically well-neutralized

- What about ion beam current?

\[ \lambda = \frac{c}{\omega_{pe}} \] collisionless electron skin depth ($n_p \sim 10^{11} \text{ cm}^{-3} \rightarrow \lambda \sim 1.7 \text{ cm}$) defines the characteristic length scale for screening current (or magnetic-field) perturbations in a cold plasma ("inductive" analog of the Debye length)

\[ \nabla \times B = \frac{\partial \mathbf{E}}{\partial t} \]

Inductive field accelerates electrons

Electron radial force balance

\[ E_r = B_{self}^\phi \frac{V_{ez}}{c} \]

Current neutralization

\[ \frac{V_{ez}}{V_b} \sim Z_b n_b / n_e < 1 \]

Magnetic pinching is dominant

\[ E_r < B_{self}^\phi \frac{V_b}{c} \]

(a) $r_b < c/\omega_{pe}$

current is not-neutralized (locally)

maximum ($j_b \times B_{self}$) self-pinching

(b) $r_b > c/\omega_{pe}$

current is well-neutralized (locally)

negligible self-pinching
There is a significant enhancement of the ion beam self-focusing effect in the presence of a weak solenoidal magnetic field (for $r_b << c/\omega_{pe}$).
Enhanced Self-Focusing is Demonstrated in Simulations

Gaussian beam: \( r_b = 0.55c / \omega_{pe} \), \( L_b = 3.4r_b \), \( \beta = 0.05 \), \( n_b = 0.14n_p \), \( n_p = 10^{10} \text{ cm}^{-3} \)

**Radial focusing force**

Central beam slice

\[ F_r/Z_b e \ (V/cm) \]

<table>
<thead>
<tr>
<th>( B_{ext} = 0 )</th>
<th>Magnetic self-pinching</th>
</tr>
</thead>
<tbody>
<tr>
<td>( B_{ext} = 300 \text{ G} )</td>
<td>Collective self-focusing</td>
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**Radial electric field**

\[ \frac{\omega_{ce}}{2\beta_b \omega_{pe}} = 9.35 \]

B_{ext} = 300 G

LSP (PIC)

The enhanced focusing is provided by a strong radial self-electric field

*Influence of the plasma-induced collective focusing on the ion beam dynamics in*

- NDCX-I is negligible
- NDCX-II is comparable to the final focusing of an 8 T short solenoid
Local Plasma Response is Drastically Different for $\omega_{ce} > 2\beta_b \omega_{pe}$ and $\omega_{ce} < 2\beta_b \omega_{pe}$

Moderate magnetic field ($\omega_{ce} > 2\beta_b \omega_{pe}$)
- $V_{e\phi} > 0$
- Radial electric field is focusing
- Diamagnetic plasma response
- Beam charge is overcompensated

Weak magnetic field ($\omega_{ce} < 2\beta_b \omega_{pe}$)
- $V_{e\phi} < 0$
- Radial electric field is defocusing
- Paramagnetic plasma response
- Beam charge is under-neutralized

$\omega_{ce} = 2\beta_b \omega_{pe}$

Resonant excitation of large-amplitude whistler waves

$n_p = 10^{11}$ cm$^{-3}$, $\beta_b = 0.05$  $B_0 = 100$G

M. Dorf et al, PoP 19, 056704 (2012).
Numerical Simulations Demonstrate Qualitatively Different Local Plasma Responses

\[ \omega_{ce} > 2\beta_b \omega_{pe} \]

- \( B_0 = 25 \) G (\( \omega_{ce}/\beta_b \omega_{pe} = 1.56 \))
- \( B_0 = 300 \) G (\( \omega_{ce}/\beta_b \omega_{pe} = 18.7 \))

- Focusing electric field
- Diamagnetic response (\( \delta B_z < 0 \))

\[ \omega_{ce} < 2\beta_b \omega_{pe} \]

- Defocusing electric field
- Paramagnetic response (\( \delta B_z > 0 \))

- \( r_b = 0.55 c/\omega_{pe} \), \( l_b = 3.4 r_b \), \( \beta = 0.05 \), \( n_b = 0.14 n_p \), \( n_p = 10^{10} \) cm\(^{-3} \)
**Whistler Wave Excitation**

Transverse magnetic field

**Schematic of the detected signal**

- **PIC (LSP)**
- **Analytic**
- **Semi-analytic**

**Signal amplitude**

- Strong wave-field excitation
- Enhanced self-focusing (local fields)

\[ \omega_{ce}/2\beta_b\omega_{pe} \]

- Beam velocity = phase velocity = group velocity

- Analytic theory is in very good agreement with the PIC simulations
- Strong wave excitation occurs at \( \omega_{ce}/2\beta_b\omega_{pe} \) (supported by PIC simulations)
- Wave-field excitations can be used for diagnostic purposes

\[ \beta_b = 0.33, \ l_b = 10r_b, \ r_b = 0.9c/\omega_{pe}, \ n_b = 0.05n_p, \ B_{ext} = 1600G, \ n_p = 2.4 \times 10^{11} \text{cm}^{-3} \]

M. Dorf et al, PoP 17, 023103 (2010).
II. The Use of Weak Magnetic Fields for Final Beam Focusing

Schematic of the present NDCX-I final focus section

**Challenges:**
- Operate 8 T final focus solenoid
- Fill 8 T solenoid with a background plasma

**Can a weak magnetic lens be used for tight final beam focusing?**
Magnetic Lens

charged particle beam $q, v_b$

magnetic lens

$v_b \times B_r$ provides azimuthal rotation ($v_\phi$)

$v_\phi \times B_0$ provides radial focusing

Conservation of canonical azimuthal (angular) momentum

$$P_\phi = mr v_\phi - \frac{q}{c} r A_\phi = \text{const}$$

$$A_\phi = \frac{r}{2} B_0$$

Cyclotron frequency

$$\Omega_c = \frac{qB_0}{mc}$$

Equation of motion

$$m \frac{d^2 r}{dt^2} = m \frac{v_\phi^2}{r} - q \frac{v_\phi B_0}{c} = F_r$$

Centrifugal force

$$F_r = - \frac{m r \Omega_c^2}{4}$$

Lorentz force
The use of a collective focusing lens reduces the required magnetic field by \((m_i/m_e)^{1/2}\).
Collective Focusing Lens (Cont’d)

Strong ambipolar electric field is produced to balance the magnetic $\mathbf{V} \times \mathbf{B}$ force acting on the co-moving electrons.

Electrons traversing the region of magnetic fringe fields acquire an azimuthal velocity of

$$V_{e\phi} = -\Omega_e \frac{r}{2} \quad \Omega_e = \frac{eB_0}{m_ec}$$

- Conditions for collective focusing
  1. Quasi-neutrality: $\omega_{pe} >> \Omega_e / \sqrt{2}$
  2. Small magnetic field perturbations: $r_b << 2c/\omega_{pe}$
  3. No pre-formed plasma inside the lens

Electric force

$eE_r = \frac{e}{c} V_{e\phi} B_0 - m_e \frac{V_{e\phi}^2}{r}$

$E_r = -m_e \Omega_e^2 \frac{r}{4e}$

$V_{e\phi} = -\Omega_e \frac{r}{2}$

$\Omega_e = \frac{eB_0}{m_ec}$
The collective focusing concept can be utilized for final ion beam focusing in NDCX.

- No need to fill the final focus solenoid (FFS) with a neutralizing plasma
- Magnetic field of the FFS can be decreased from 8 T to ~700 G
Numerical Simulations Demonstrate Tight Collective Final Focus for NDCX-I

Schematic of the NDCX-I simulation R-Z PIC (LSP)

Beam injection parameters:

\[ \text{K}^+ \text{ @ } 320 \text{ keV, } r_b=1.6 \text{ cm, } \]
\[ I_b=27 \text{ mA, } T_b=0.094 \text{ eV, } \]

Plasma parameters (for the simulation II)

\[ n_p=10^{11} \text{ cm}^{-3}, T_e=3 \text{ eV} \]

Beam density @ focal plane

Radial electric field inside the solenoid

Beam current @ focal plane

M. Dorf et al, PoP 18, 033106 (2011)
Conclusions

Even a weak magnetic field (several hundred gauss) can have a significant influence on neutralized ion beam transport.

Self-focusing force can be significantly enhanced by application of a weak magnetic field (~100 G) for $r_b << c/\omega_{pe}$.

- Can be important for the design of a heavy-ion driver (e.g. NDCX)
- Can be utilized for self-pinch ion beam transport applications
- Can be utilized for the ion beam final focus (e.g. NDCX-I, II)
- Perhaps could be utilized for collimation of laser generated proton beams

For a given focal length the magnetic field required for a neutralized beam is smaller by a factor of $(m_e/m_i)^{1/2}$. 
Enhanced Self-Focusing VS Collective Lens

**Enhanced self-focusing**

\[ F_{self} = Z_b^2 m_e V_b^2 \frac{1}{n_e} \frac{dn_b}{dr} \]

Non-linear force can effectively balance \( \nabla p \sim T_b \nabla n \) (important for self-pinch transport)

**Conditions:**
\[ \frac{\omega_{ce}}{2\beta_b \omega_{pe}} \]
\[ r_b \gg r_{ge} = \frac{V_b}{\omega_{ce}} \sqrt{1+\frac{\omega_{ce}^2}{\omega_{pe}^2}} \]
\[ n_p = 10^{11} \text{ cm}^{-3}, \beta_b = 0.05 \]
\[ B_0 > 100 \text{ G} \]
\[ r_b > 1 \text{ cm} \]

**Collective Lens**

neutralized ion beam

\[ F_{col} = -\frac{r}{4} m_i \Omega_e \Omega_i \]

Linear force (important for beam focusing)

**Conditions:**
\[ \omega_{pe} \gg \Omega_e / \sqrt{2} \]
\[ r_b \ll 2c/\omega_{pe} \]
\[ \text{no plasma inside the lens} \]

In the limit of \( r_b \sim r_{ge} \) and \( Z_b n_b \sim n_e \) \( \rightarrow F_{self} \sim F_{col} \)